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EVALUATION OF AN AUTOMATIC SELECTION METHODOLOGY OF MODEL PARAMETERS FROM STABILITY DIAGRAMS ON A DAMAGED BUILDING

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ABSTRACT

Automatic modal parameter estimation is necessary for continuous structural health monitoring and damage assessment. The interpretation of stabilization diagrams is the most used tool for parametric system identification. In the present article an automated interpretation of the stability diagram is described and applied upon five years of vibration data of a building structure located in Chile which has been subjected to more than 1700 earthquakes. The automated method considers soft and hard validation criteria and three types of clustering algorithms. Results obtained from its application indicate that they are highly dependent on both the quantity and quality of the sensors, and several notes to achieve a fully functional automatic procedure are exposed.

Keywords: Structural health monitoring, modal analysis stabilization diagrams, modal validation criteria, structural dynamics.

1. INTRODUCTION

Continuous structural health monitoring and damage assessment requires an automatic determination of modal parameters. The most used tool for (manual) modal analysis based on parametric system identification is the interpretation of stabilization diagrams. Therefore an automated interpretation of such diagrams is described and applied in this paper.

Parametric system identification methods require defining the model order. The model order is an integer value that equals the number of eigenvalues present in the model, and in theory, the double of modes determined by the identification method. Given that specifying a larger or smaller model order, and even specifying the most suitable model order generates spurious modes (i.e. modes that do not represent a valid vibration mode of the structure in analysis), the stabilization diagram is used to determine which ones of the identified modes are valid or physical modes.

The stabilization diagram can be roughly defined as a plot of model order vs. eigenfrequencies for a wide range of model orders. From the empirical observation of numerous modal identification problems, the physical modes appear at nearly the same frequency when the model order is overspecified, while the spurious modes do not. Thus the use of the stabilization diagram is to interpret which of the modes are physical (or stable) and, from each of these stable modes, to determine a representative mode.

To successfully interpret these diagrams is often not straightforward, and additional criteria may be necessary to implement, hence requiring a high amount of user interaction and judgement by the analyst, hence justifying the need to automate this interpretation.

Covariance-driven stochastic subspace identification (SSI-COV) [1] is used in order to generate the stabilization diagram, considering an operation modal analysis framework.

The aim of this paper is to evaluate an automated modal selection based upon the interpretation of the stabilization diagram by applying it on a building structure. The analysed structure vibrations have been continuously measured from 2009 along with the environmental conditions.

The organization of this paper is as follows. The evaluated method is presented on section 2. Section 3 presents the main properties of the damage building along with the obtained results from applying this automation method for five years (2009-2014). Finally the conclusions are presented in section 4.

2. AUTOMATIC INTERPRETATION OF STABILIZATION DIAGRAMS

The used method is heavily based on the work developed by Reynders et al. [2] with a modification of the final stage. The method can be summarized as follows:

Stage 1: The modes from the stabilization diagram are initially partitioned using k-means clustering algorithm (k=2) upon soft single-mode validation criteria, resulting in two clusters: one with (possibly) physical modes, and one with spurious modes. The modes assigned to the spurious cluster are deleted from the analysis, and the resulting physical modes are “cleaned” further by the use of hard validation criteria.

Stage 2: From the set of modes resulting from the previous stage, similar modes are grouped together using a hierarchical clustering algorithm considering both the distance and cut-off distance of the algorithm as a function of the eigenvalues and MAC values.

Stage 3: The resulting sets of similar modes formed in the previous stage are partitioned using k-means clustering algorithm (k=2) upon the number of elements containing each set. Thus assigning each set as physical or spurious (in which case they are deleted from the analysis).

Stage 4: From each set of similar modes assigned as physical, a single mode is chosen. In order to do this a density based clustering method (DBSCAN) is used.

This process does not require any user-defined parameter to be initially adjusted, and thus ensures a fully automated analysis.

2.1. Single Mode Validation Criteria

Several single-mode validation criteria which apply to SSI-COV are presented in this section. They are divided on hard and soft validation criteria. Soft validation criteria are further categorized as distance, mode shape and energy criteria, which in order to give the same weight for the k-means clustering method used, are normalized in the range [0,1]. Hard validation criteria will be tests applied to each mode that will yield a binary answer {0,1}.

2.1.1. Distance validation criteria

Distance validation criteria are determined by either the eigenfrequencies f_i^k, damping ratios β_i^k, continuous-time eigenvalue $\lambda_{c_i}^k$ or mode shape $\boldsymbol{\phi}_i^k$. The superscript is associated to the model order and the subscript to each mode identified by each model order.

Note that (theoretically) the continuous-time eigenvalue is a function of the eigenfrequency and damping ratio (see, for example, [3]):

$$\lambda_{c_i} = -\beta(2\pi f_i) \pm j(2\pi f_i)\sqrt{1-\beta_i^2} \quad (1)$$

Distance criteria determined by eigenfrequencies $d(f_i^k)$, damping ratios $d(\beta_i^k)$, and continuous-time eigenvalue $d(\lambda_i^k)$ distance criteria can be generally defined by:

$$d(\chi_i^k) = \min\left(\frac{|\chi_i^k - \chi_j^{k-1}|}{m \quad (|\chi_i^k|, |\chi_j^{k-1}|)}\right) \quad (2)$$

Where χ_i^k represents either f_i^k, β_i^k or $\lambda_{c_i}^k$. In words, this parameter corresponds to the minimum of the (normalized) distance between the mode in analysis and the modes associated to the next inferior model order. In the case of physical stable mode, this parameter would take a low value (limited to 0) given the probability of encountering a similar mode in the next inferior model order, and a high value for spurious unstable mode (limited to 1).

Distance criteria determined by the mode shape $\boldsymbol{\phi}_i^k$ is obtained using the modal assurance criteria (MAC) value:

$$M \quad (\boldsymbol{\phi}_i, \boldsymbol{\phi}_j) = \frac{|\boldsymbol{\phi}_i^{\square}\boldsymbol{\phi}_j|^2}{\|\boldsymbol{\phi}_i\|_2^2 \|\boldsymbol{\phi}_j\|_2^2} \quad (3)$$

The distance criteria as a function of the MAC value of the mode shape $\boldsymbol{\phi}_i^k$ is then defined as:

$$M \quad C_d(\boldsymbol{\phi}_i^k) = \max\left(M \quad (\boldsymbol{\phi}_i^k, \boldsymbol{\phi}_j^{k-1})\right) \quad (4)$$

Analogous to the other distance criteria, this value corresponds to the maximum of the MAC values between the mode in analysis and the modes associated to the next inferior model order. The observations are similar to the other distance criteria, considering high MAC value (limited to 1) to correlated (similar) mode shapes, and low MAC values (limited to 0) to spurious ones.

2.1.2. Mode shape validation criteria

Two validation criteria based upon the mode shape complexity are exposed: the modal phase collinearity (MPC) and the mean phase deviation (MPD). For proportionally damped structures the mode shapes lie in a straight line in the complex plane, and as such the mode shape is considered as a real or monophasic vector. Both criteria aims to quantify the degree of monophasic behaviour of the mode shape vector, but given that this is not always the case, they should be used with care.

The MPC value is defined as [4]:

$$M \quad (\phi_j) = \left[\frac{2\lambda_1}{\lambda_1 + \lambda_2} - 1\right]^2 \quad (5)$$

Where λ_1 and λ_2 are the maximum and minimum eigenvalue (respectively) of the covariance matrix S_c between the real and imaginary part of the mode shape:

$$\boldsymbol{S}_c = \begin{bmatrix} R \quad (\boldsymbol{\phi}_i)^T R \quad (\boldsymbol{\phi}_i) & R \quad (\boldsymbol{\phi}_i)^T I \quad (\boldsymbol{\phi}_i) \\ R \quad (\boldsymbol{\phi}_i)^T I \quad (\boldsymbol{\phi}_i) & I \quad (\boldsymbol{\phi}_i)^T I \quad (\boldsymbol{\phi}_i) \end{bmatrix} \quad (6)$$

If the covariance matrix contains only one eigenvalue different from zero MPC would take a value equal to 1 (perfect collinearity), in any other case the collinearity would be imperfect, so two non-negative eigenvalues would be present diminishing the MPC value. The limit in which the real and imaginary part presents no collinearity, the eigenvalues would be equivalent ($\lambda_1 = \lambda_2$) so MPC would take a value of zero.

The MPD of the mode shape is determined by [2]:

$$M \quad (\boldsymbol{\phi}_j) = \frac{\sum_{o=1}^{n} \mathrm{w_o} \arccos \left| \frac{R \ (\phi_{j_o}) V_2 \ - I \ (\phi_{j_o}) V_1}{\sqrt{V_1^2 \ + V_2^2} \left| \phi_{j_o} \right|} \right|}{\sum_{o=1}^{n} w_o}, \quad \sum_{o=1}^{n} w_o \neq 0 \tag{7}$$

Where the subscript o is used to define each element of the mode shape vector, w_o are weighting factors that are chosen equal to $\left|\phi_{j_o}\right|$ (to provide higher weight to higher amplitude mode shape elements), and V_2 , V_1 are the (2,2) and (1,2) element of the V matrix determined from the singular value decomposition of the matrix formed by the real and imaginary part of the mode shape vector:

$$\boldsymbol{U} \ \ \boldsymbol{V}^t = \left[R \ (\boldsymbol{\phi}_j) \quad I \ (\boldsymbol{\phi}_j)\right] \tag{8}$$

Finally, given that for the appropriate use of the k-means algorithm associated with the validation criteria, this value is normalized by 45°. Hence, if the mean phase is wide (limited by 45°) then MPD would take a value near unity, and in case of equal phase for each element of the mode shape vector, the MPD would take a value of zero.

2.1.3. Energy validation criteria

Two energy validation criteria are exposed: modal transfer norm (MTN) and the contribution of each mode to the measured response (CMM). Both aims to measure the energy associated to each mode.

The MTN is defined as the maximum of the set of singular values of the modal decomposition of the positive power spectral density evaluated at each eigenfrequency $S_{y \ ,j}^{+}(\omega_j)$ [5], [1]:

$$M \quad (j) = \max \sigma \left(\boldsymbol{S}_{y \ ,j}^{+}(\omega_j) \right) \tag{9}$$

Where $\sigma(\cdot)$ represents the set of singular values. This criterion measures the error that is made when the mode j is removed from the model; therefore high values would usually represent physical modes, and near zero values spurious ones. In order to limit the given value to the range [0,1], this criterion is normalized by the maximum value for each model order.

The CMM is defined as [6]:

$$C \quad (j) = \frac{1}{l} \sum \boldsymbol{\Delta}_{m_j} \tag{10}$$

Where $\boldsymbol{\Delta}_{m_j}$ is a vector formed by the diagonal elements of the matrix $(\boldsymbol{Y}^{m_j} \boldsymbol{Y}^T)$, where $\boldsymbol{Y}$ is the extended output matrix and $\boldsymbol{Y}^{m_j}$ is the extended modal estimated output matrix:

$$\boldsymbol{Y} = [\boldsymbol{y}_1 \, \boldsymbol{y}_2 \cdots \boldsymbol{y}_N], \quad \boldsymbol{Y}^{m_i} = \left[\boldsymbol{y}_1^{m_i} \, \boldsymbol{y}_2^{m_i} \cdots \boldsymbol{y}_N^{m_i}\right] \tag{11}$$

The extended output matrix $\boldsymbol{Y}$ is formed directly from the output signals of the structure, and the extended modal estimated output matrix is determined from the innovation (Kalman filter) model in modal form. For more details refer to [6].

The CMM is, as implied by its name, the contribution of each mode to the total (measured) response. As seen in [6], the CMM values are limited by the quantity of noise present in the signal; therefore, in order to give the appropriate weight for its use in the k-means clustering method, this criterion is normalized by the maximum value for each model order.

2.1.4. Hard validation criteria

Hard validation criteria are binary values representing the results of a test (if it passes the test it is assigned a value of 1, and if it does not, it gets a value of 0). According to Reynders [2] five hard validation criteria are used:

1) Presence of negative or high damping ratios ($\beta_i > 0[\%], \beta_i < 20[\%]$). Given that structures are always stable, damping ratios of physical modes should always be positive, and given that high damping ratios are unlikely to occur, its value is limited to 20[%].
2) Presence of complex conjugate pairs. For every physical mode of a structure with continuous-time eigenvalue λ_{c_i} and mode shape $\boldsymbol{\phi}_i$, a second mode with conjugate properties (for each property) should also be present. Hence the presence of complex conjugate pairs for λ_{c_i} and $\boldsymbol{\phi}_i$ (independently) indicates a (possibly) physical mode.
3) Frequency limit. A limit in the eigenfrequency value is applied in order to limit the search of physical mode to a range of interest. For the analysed structure only frequencies under 10 [Hz] are considered relevant.

2.2. First Stage

The first stage of the automatic interpretation method consists of two parts: the k-means clustering method is applied as a function of the soft validation criteria, and then the compliance of all of the hard validation criteria are verified. The resultant set of modes is the result of this stage (those who are classified as physical modes by the clustering method, and at the same time meets all of the hard criteria).

The k-means (k=2) clustering algorithm minimizes the sum of the squared Euclidian distance between each mode defined by a point p_i □ R^{n_s} formed by the values taken by each mode for the soft validation criteria. The implementation of this algorithm is shown in [2].

2.3. Second Stage

The second stage groups together similar modes (the set of modes that results from the previous stage) by means of an agglomerative hierarchical clustering (details of its implementation is shown on [2]). The distance function between modes considered in this clustering algorithm is defined as:

$$d(i,j) = \frac{\left|\lambda_{c_i} - \lambda_{c_j}\right|}{m \quad \left(\left|\lambda_{c_i}\right|, \left|\lambda_{c_j}\right|\right)} + 1 - M \quad (\boldsymbol{\phi}_i, \boldsymbol{\phi}_j) \tag{12}$$

The cut-off distance used to limit the formation of groups is defined as:

$$\tilde{d} = \mu_{s1} + 2\sigma_{s1} \tag{13}$$

Where μ_{s1} and σ_{s1} are the mean and standard deviation of the vector s_1 formed by the soft validation criteria of continuous-time eigenvalue distance and MAC distance of the set of modes that meets the conditions of stage one:

$$\boldsymbol{s_1} = d(\lambda)_1 + 1 - d \qquad (\boldsymbol{\phi})_1 \tag{14}$$

The assignation of each mode into a set of similar modes is the result of this stage.

2.4. Third Stage

Based upon the stabilization property of physical modes, the number of elements of a group (defined in the previous stage) should be larger for physical (stable) modes than spurious (unstable) modes. From this perspective 2-means clustering algorithm is applied as a function of the number of elements of each group (details in [2]) in order to classify each group of similar modes as physical or spurious.

Given that the number of groups determined could be all physical, an additional number of empty groups are created equal to the number of groups that has at least a fifth of the number of elements present in the larger group.

In consequence, the k-means clustering algorithm is applied as a function of the number of elements of the groups determined from the previous stage and the empty groups created. The groups classified as groups of physical modes are the result of this stage.

2.5. Fourth Stage

The fourth and final stage aims to define a representative element of each group (or stable mode) formed in the previous stage. Reynders et al. [2] presented three possible ways to find this element: from the mode with the median damping value, highest MPC value or lowest MPD value. Due to some anomalous values, obtained with these three methods, a new method to determine this mode was introduced.

The methodology objective is to detect the densest cluster for each group on the frequency v/s damping plane by means of the density-based spatial clustering algorithm (DBSCAN, [7]). The DBSCAN algorithm depends on two parameters which can be roughly defined as: the minimum number of elements k required for a cluster to be considered as such (instead of noise) and the maximum distance ϵ for any given element to seek close elements. In order to maintain the automatic feature of the interpretation method, k will be considered as 1 and the distance ϵ will be defined as the necessary distance to get k elements given a uniform distribution of the elements [8]:

$$\epsilon = \sqrt{\frac{V \ \Gamma\left[\frac{n}{2}+1\right]}{m\sqrt{\pi^n}}} \tag{1}$$

The methodology used to detect the representative element for each group is the following:

1) Each mode is classified using DBSCAN from the frequency-damping plane.
2) Every mode that is not assigned to the biggest cluster formed (as a function of the number of elements present in it) is discarded.
3) Steps 1 and 2 are repeated until no more clusters are formed (only noise would be present). The representative mode will be the one closest to the mean (frequency and damping) of the final cluster determined.

The previous methodology ensures the choosing of a mode centered on the densest cluster. This stage will result in the construction of a set of representative modes of the structure in study, therefore finalizing the process.

3. CASE STUDY

3.1. Damage building and identification considerations

The damage building analysed is the "Torre Central" building of the Mathematics and Physics Sciences Faculty of the University of Chile, located in Santiago, Chile. The building has eight stories and two basements, an approximate height of 30.2 [m] above ground level and is structured based on shear walls 35 [cm} thick and slabs 25 [cm] thick.

For measuring the vibration of the structure, eight uniaxial accelerometers are used, two of them located on the basement, and three on the third and eighth floor. Environmental sensors are also present on the structure.

The identification considers 15 minutes length records from June of 2009 to July 2014 with a variable sample frequency between 100-200 [Hz] which is resampled to 25 [Hz]. Channels located on the basement are deleted from the analysis given that they represent the vibrations of the soil rather than the structure itself. Therefore only six channels where used.

The identification process is made through the SSI-COV algorithm [1] and a fast multi-order computation algorithm developed by Dölher et al. [9].

The Fig. 1 shows an example of the initial stabilization diagram and the resulting diagram following the full application of the automated interpretation process. Note that to lower the computational costs only 51 model orders are computed in the range [50,150]. The lower limit is established so as to ensure the presence of all stable modes.

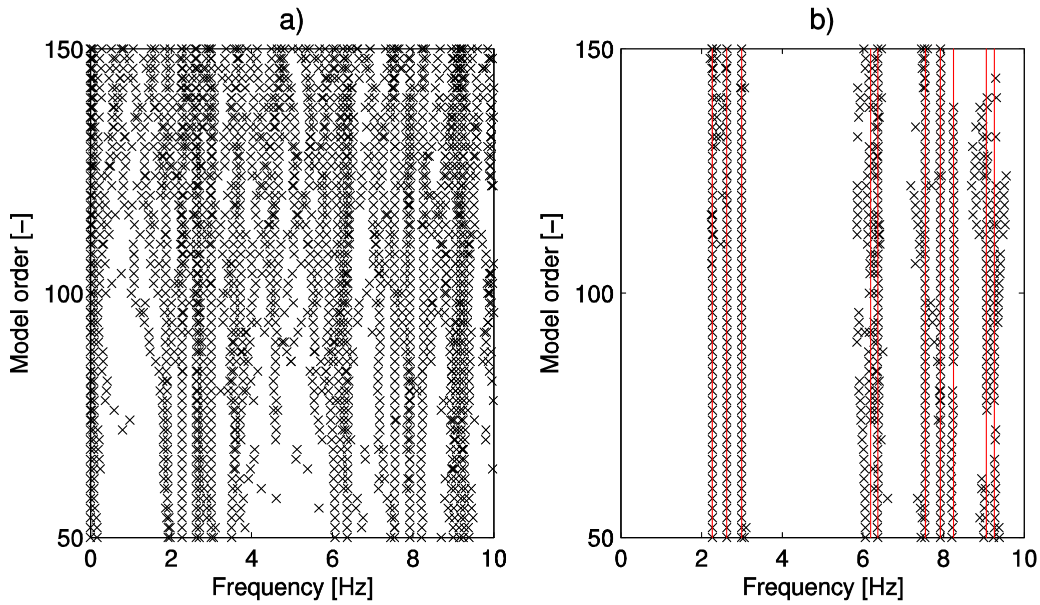

Figure 1. Stabilization diagram a) initial condition, b) processed diagram

3.2. Results and observations

It has been observed that a high quantity of the records presented high level of noise and a large quantity of them also presented seismic data which affects is notby the SSI-COV algorithm. Thus a lower number of channels are used so as to ensure a higher quality of the identification process, although not completely eliminating the occurrence of these high noise level or seismic data... The presence of these faulty signals provides interesting information to the obtained results.

In order to analyse the efficiency of the soft validation criteria the difference between centroids of the k-means algorithm was calculated. This difference represents the weight that each criterion holds on the partition between physical and spurious modes. A higher (absolute) value indicates a higher weight, and a negative value indicates that the physical centroid is closer to the ideal spurious value and inversely for the spurious centroid, hence illustrating poor quality of a given criterion.

Figure 2 shows the centroid difference histogram for each soft validation criterion, along with the mean value. Notice that using a lower number of channels the expected normal distribution is altered affected the expected normal distribution for the $M \quad C_d$ and M criterion given the lower quality of these parameters when using a lower number of channels. From this figure a negative value can be observed for the eigenvalues distance $d(\lambda_c)$, and a low value for the $M \quad C_d$ thus showing a poor quality of these classification criteria under this circumstances. Due to the low discrimination capacity the MAC, eigenvalue, frequency and damping distance criteria only eigenvalue and MAC distance criteria were preserved for hierarchical clustering.The poor quality of the distance validation criteria gave reason to omit the other two validation criteria from the process.

Higher difference values, better discrimination, where obtained from the mode shape complexity criteria, thus illustrating a better quality.

The high level of noise of the system resulted in poor quality indicators for the energy based criteria. Although filtering of some of the noise is possible and could improve the efficiency of these criteria, the computation effort associated of implementing the alternative data-driven stochastic subspace identification (SSI-DATA) algorithm [1] (necessary for the application of these criteria) in addition to the higher computational effort associated to obtain these values, justifies in this case the omission of them from the analysis.

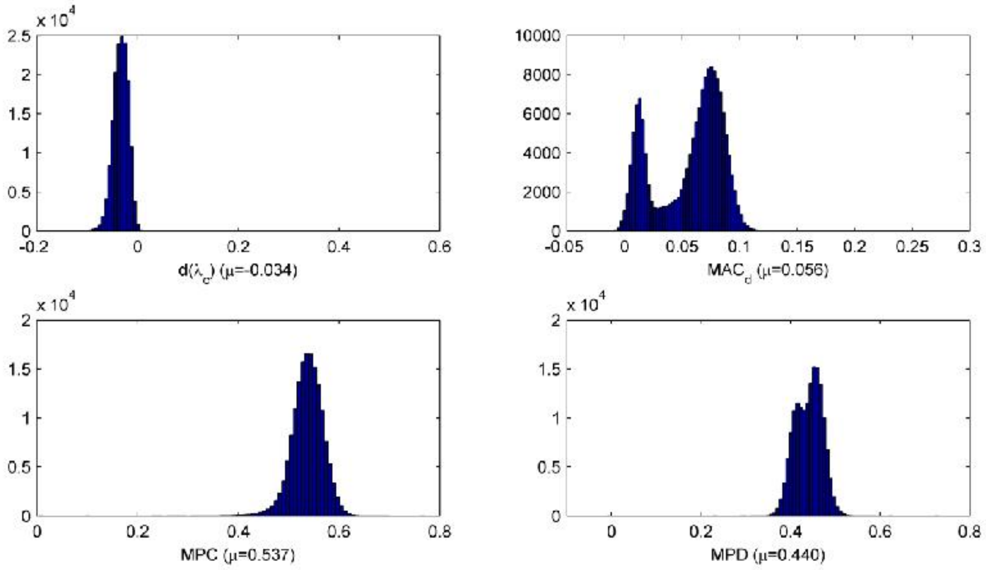

Figure 2. Centroid difference histogram

The second stage groups together similar modes, the precision of the formed groups will determine the quality of the results provided by stage three. Therefore it has been observed that when the hierarchical algorithm of stage two generates erroneous groups: either by associating additional spurious modes to stable columns groups, or forming large groups of spurious modes, stage three will also generate errors. The first case generates errors in stage three due to a higher weight to the large groups formed, thus possibly eliminating physical groups when using k-means clustering in the third stage. The second case will give higher weight to a spurious group, hence the possibility of considering it as a physical group. Both cases then are dependent on the hierarchical clustering considered, and in particular, the distance and cut-off distance considered. Therefore the results from the application of the third stage where as accurate as the clustering of stage two, and it proved to be an efficient way of distinguishing between physical stable modes and spurious modes when good quality signals where used.

The proposed density-based method to select the representative element provides an intuitive approach based on the observation of the clusters formed in the frequency-damping plane, therefore giving accurate results.

Finally, a simple modal tracking was implemented on the obtained results. The methodology considers boundaries on frequency and damping ratio for each mode. Manually setting these thresholds manages to delete additional spurious modes identified. Other strategies are being developed. Figure 3 shows the frequencies determined by the automatic method, and figure 4 shows the results from the modal tracking process. Both cases considers only the year 2010.

From these results a sudden change on the frequencies can be observed, which occurs after the magnitude 8.8 Chilean earthquake of 2010. This change reflects damage in the structure. Additionally lower precision is detected for higher frequency modes, thus reflecting a lower quality on the identification process and is also result from the simplistic methodology used for modal tracking. In spite of the simplicity of the used methodology, the precision of the results is remarkable, where even better results could be obtained by means of the use of MAC in the modal tracking, or a more robust method.

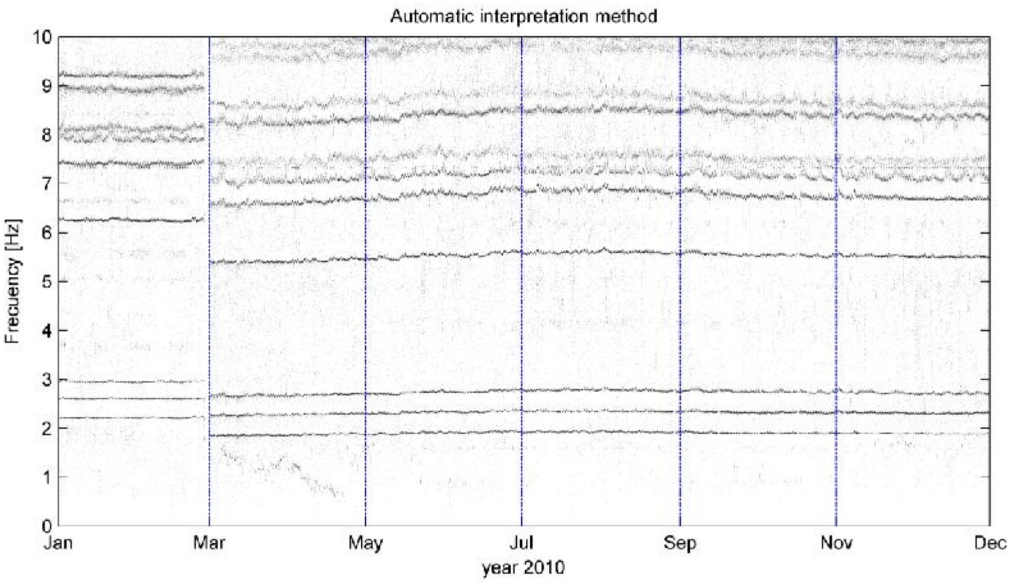

Figure 3. Frequencies identified (2010)

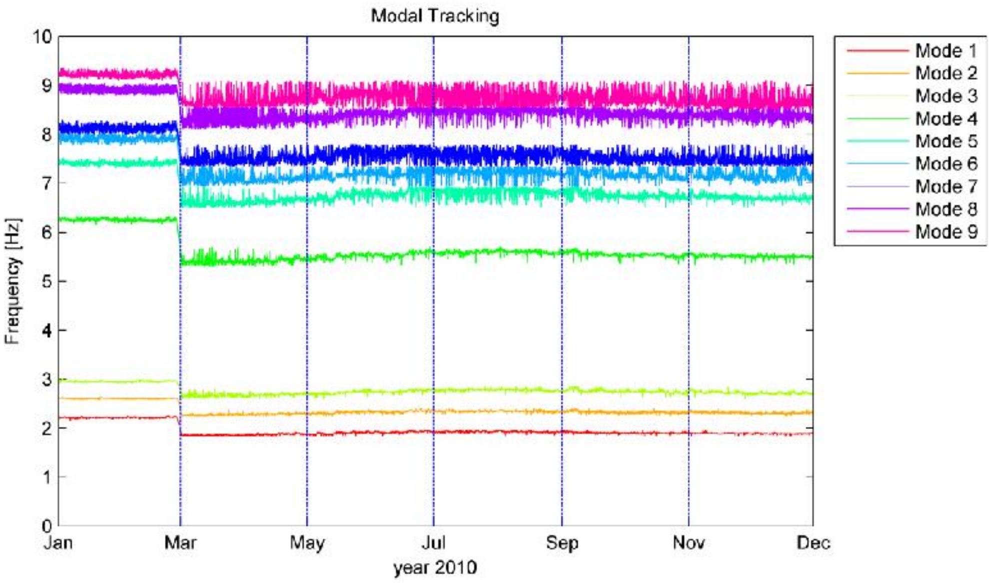

Figure 4. Modal Tracking (2010)

The results obtained from the four year analysis shows that relatively stable spurious modes (high number of elements inside the determined groups) can be encountered when low quality signals are used, and thus increasing the importance to effectively distinguish similar modes (stage two). The quality of the results obtained from stage three and four are a direct consequence of the efficiency of this stage, which in turn depends on the results of the first stage in terms of detecting and clearing the highest possible amount of spurious modes and the lowest possible amount of physical ones.

4. CONCLUSIONS

The fully automated method was evaluated over the four year span of records from the damage building. Hard and soft validation criteria where applied and the results indicate a higher efficiency of the hard validation criteria and a lower quality from the clustering based upon the soft validation criteria. This indicate that hard validation criteria proves to be more useful in the detection of stable columns, therefore soft validation criteria should be limited to pre-defined values in order to make them hard validation criteria, and thus providing better quality results in the first stage, this can be easily applied to the efficient mode shape criteria. Although this requires more user-defined parameters, these can be easily adjusted to successfully apply to the four year analysis.

The process to detect the stable columns (stage two) could be further improved based on hierarchical clustering as a function of MAC in order to discriminate near frequency modes using a user-defined cut-off distance. Notice that a higher quality would be attained from a higher number of sensors. Both stage three and four should work efficiently as long as the stage two results are accurate.

Modal tracking was successfully and easily implemented on the results, thus establishing a continuity of the identified modes, and also managing to delete spurious modes that are determined as physical modes by the automatic method. In spite of these results, a more robust, MAC-based modal tracking would yield better results.

The quality and quantity of sensors used is therefore a determining factor in the precision of the automatic method, given that it would improve all mode shape dependant parameters such as MAC distance or mode shape criteria.

Computational time required for completely processing each record is roughly 3 [s] (on a Intel(R) Core(TM) i7 2640M CPU @ 2.80GHz processor, 8.00 GB ram) in lieu of the high quantity of processes involved, which proves to be a time-efficient method.

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