First European Conference on Earthquake Engineering and Seismology
(a joint event of the 13th ECEE & 30th General Assembly of the ESC)
Geneva, Switzerland, 3-8 September 2006
Paper Number:

NUMERICAL CODE FOR SEISMIC ANALYSIS OF STRUCTURES INCORPORATING ENERGY DISSIPATING DEVICES

Pablo MATA A[1], Sergio OLLER[2], Alex H. BARBAT[3] and Ruben BOROSCHEK[4]

SUMMARY

The nonlinear dynamic response of civil structures with energy dissipating devices is studied. The structure is modeled using the Vu Quoc–Simo formulation for beams in finite deformation. The effects of shear stresses are considered, allowing rotating the local system of each beam independently of the position of the beam axis. The material nolinearity is treated at material point level with an appropriated constitutive law for concrete and fiber behavior for steel reinforcements and stirrups. The simple mixing theory is used to treat the resulting composite. The equation of motion of the system as well as the conservation laws are expressed in terms of sectional forces and generalized strains and the dynamic problem is solved in the finite element framework. A specific kind of finite element is proposed for modeling the energy dissipating devices. Several tests were conducted to validate the ability of the model to reproduce the nonlinear response of concrete structures subjected to earthquake loading.

1. INTRODUCTION

In the traditional approach to earthquake engineering design, the computations are carried out on the basis of linear elastic static analysis. The nonlinear behavior and energy dissipation can be accounted for in a trivial manner by a force-based approach, analyzing the elastic response spectra of a single degree of freedom system, *SDF*, and the so called reduction factor method for introducing the material ductility. A more rational concept, the displacement based approach, turns toward the design based on the critical limit states of the elements [Davenne et.al. 2003]. Both design procedures are based on the study of a *SDF* system or simplified substitute structures, which are not capable to account for the load redistribution inside the structures due to local non-linearity. This is one of the major drawbacks preventing a realistic description of the global and local behavior of the structure up to the failure [Davenne et.al. 2003]. By other hand, additional improvements can be done if energetic concepts are taken into account. The passive control of structures takes advantages of the possibility of dissipate energy in specific devices alleviating the stresses in the main structural elements and controlling the lateral displacements of the whole structure.

One choice to perform realistic analysis of structures equipped with energy dissipating devices for seismic loading is employing nonlinear time history analysis assuming physical descriptions for the materials and applying transient loadings on the structure in terms of natural or simulated ground motions. The model employed for the structure should be able to simulate the changes of configuration of the structure during the dynamic action, especially for the case of flexible structural behavior. Additionally, appropriated constitutive

[1] Technical University of Catalonia.Edificio C1, Campus Norte UPC. Gran Capitán s/n. Barcelona 08034, Spain
Email: pmata@cimne.upc.edu,
[2] Technical University of Catalonia.Edificio C1, Campus Norte UPC. Gran Capitán s/n. Barcelona 08034, Spain
Email: sergio.oller@upc.edu
[3] Technical University of Catalonia.Edificio C1, Campus Norte UPC. Gran Capitán s/n. Barcelona 08034, Spain
Email: alex.barbat@upc.edu
[4] University of Chile, Civil Engineering Department., Blanco Encalada 2002. Santiago, Chile.
Email: rborosch@ing.uchile.cl

laws have to be provided for the materials of the elements and for dissipating devices.

In this work the latter is achieved modeling the structure by mean of employing the Simo-Vu Quoc formulation for beams and rods capable of undergoing large strains and displacements [Simo et.al. 1985]. Each beam section is meshed into a grid of cells, each of them corresponding to a fiber directed along the beam axis. The material associated with a fiber can be composed by several components, employing the simple mixing theory for the treatment of the resulting composite [Oller, et.al. 1997]. The incorporation of energy dissipating devices is obtained developing a special rod element with only one integration point. Appropriated one-dimensional constitutive laws or strain–stress relationships are provided for the element. Numerical examples are studied to validate the ability of the model for simulating the dynamic response of structures subjected to seismic loading.

2. KINEMATIC OF BEAMS IN FINITE STRAIN

Nonlinear analysis of three-dimensional beam–like structural systems subjected to very large displacements is a problem frequently encountered in earthquake engineering. The 3D beam formulation employed here makes use of the equilibrium equations in terms of stress resultants in order to deduce on the energy conjugate strain measure through application of the virtual work principle. The Newmark method is employed for integrating the equations of motion in the dynamic linearized version [Simo et.al. 1985, 1986; Ibrahimbegovic, 1995].

2.1 Kinematics' Description

A typical cross section of the beam will be associated with a orthonormal basis vector, $\{t_i(S,t)\}_{i=1,2,3}$ of a moving frame attached to its centroid, where $S \in [0,L] \subset R$ denotes the curvilinear coordinate along the line of centroids of the undeformed beam, $t \in R$ is a time parameter. The $t_3(S)$ component remains normal to the section all time. The fixed (so called material description) reference axis of the same section is denoted by $\{E_i(S,0)\}_{i=1,2,3}$ so that $\{t_i(S,0)\}_{i=1,2,3} \equiv \{E_I(S)\}_{I=1,3} \forall S \in [0,L]$. The fixed spatial basis is denoted by $\{e_i(S,0)\}_{i=1,2,3}$. See figure 1. The orientation of the moving frame $\{t_i(S,t)\}_{i=1,2,3}$ along $S \in [0,L]$, and through time $t \in R$ is specified by an orthogonal transformation $\Lambda(S,t) = \Lambda_{iJ}(S,t) e_i \otimes E_J$ such that $t_I(S,t) = \Lambda(S,t)E_I = \Lambda_{iJ}(S,t)e_i$. The position $x_0 \in R^3$ of the centroid of the cross section is defined by the map: $x_0 = f_0 = f_{0i}(S,t)e_i$. Here $\Lambda(S,t) \in SO(3)$ the special orthogonal (Lie) group having the following property: $\Lambda\Lambda^t = 1$. The derivatives of the orthogonal transformation are summarized in the following formulas [Simo et.al. 1985, 1986]:

Derivatives of the moving frame

Spatial	Material	
$\frac{\partial \Lambda(S,t)}{\partial S} = \Omega(S,t)\Lambda(S,t)\,;\; \frac{\partial \Lambda(S,t)}{\partial t} = W(S,t)\Lambda(S,t)$	$\frac{\partial \Lambda(S,t)}{\partial S} = \Lambda(S,t)K(S,t)\,;\; \frac{\partial \Lambda(S,t)}{\partial t} = \Lambda(S,t)\overline{W}(S,t)$	(1)
$\Omega = \begin{bmatrix} 0 & -w_3 & w_2 \\ w_3 & 0 & -w_1 \\ -w_2 & w_1 & 0 \end{bmatrix}$	$K = \begin{bmatrix} 0 & -k_3 & k_2 \\ k_3 & 0 & -k_1 \\ -k_2 & k_1 & 0 \end{bmatrix}$	(2)
$w = w_1 e_1 + w_2 e_2 + w_3 e_3 = k_1 t_1 + k_2 t_2 + k_3 t_3$	$k = k_1 E_1 + k_2 E_2 + k_3 E_3$	(3)
$w = w_1 e_1 + w_2 e_2 + w_3 e_3 = \overline{w}_1 t_1 + \overline{w}_2 t_2 + \overline{w}_3 t_3$	$w = \overline{w}_1 E_1 + \overline{w}_2 E_2 + \overline{w}_3 E_3$	(4)

where $\Omega(S,t), W(S,t), K(S,t) \overline{W}(S,t)$ are the spatial and material representation of the curvature tensors and spins of the cross section respectively. Note that $K + K^t = 0$, $\Omega + \Omega = 0$.

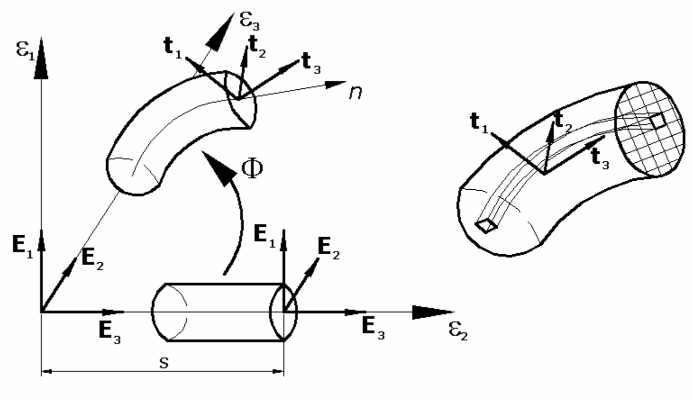

Figure 1: Kinematics of beams in finite strain for fiber sections.

2.2 Stress Resultant and Couples

Denoting by $P = T_I \otimes E_I$ the non symmetric first Piola–Kirchhoff stress tensor, the stress resultant n and the spatial stress couple m over a cross section $\Gamma \subset R^2$ in the current configuration, are defined according with the equation (5) as:

$$n = \int_\Gamma T_3 d\Gamma, \qquad m = \int_\Gamma (x - f_0(S,t)) \times T_3 d\Gamma \tag{5}$$

The material version of the stress resultants and couples are obtained pulling back n, m to the reference configuration by mean of Λ [Simo et.al. 1985]. Appropriated strain measurements conjugated to the corresponding stress resultant and couples are obtained trough the *stress power equivalence* [Simo et.al. 1986], equation (6).

$$\int_{\Gamma\times[0,L]} P : \dot{F} d\Gamma dS = \int_{[0,L]} [n \cdot \overset{\nabla}{g} + m \cdot \overset{\nabla}{w}] dS = \int_{[0,L]} [N \cdot \dot{\Gamma} + M \cdot \dot{k}] dS \tag{6}$$

where $\dot{F}$ is the time derivative of the deformation gradient and a superimposed dot denotes time differentiation. Here, $\overset{\nabla}{(\bullet)} = (\partial/\partial t)(\bullet) - w \times (\bullet)$ denotes the *co-rotated rate*, that is, the rate measured by an observer attached to the moving frame. The corresponding strain measurements are given by the equation (7).

$$g = \partial f_0(S,t)/\partial t - t_3; \quad \Gamma = \Lambda^t \, \partial f_0(S,t)/\partial t - E_3; \quad k = \Lambda^t w \tag{7}$$

2.3 Weak Form: Inertia Operator

The system of partial differential equations to be solved consists of the balance laws and constitutive equations expressed in local form. Considering any arbitrary admissible variation $h(S) = (h_0(S), J(S))$ of the spatial configuration of the rod (j, Λ), and multiplying them by the local form of the balance laws, after several mathematical procedures, it is possible to obtain the weak form of the equilibrium equations (for a detailed description consult Simo et.al. 1986, 1988):

$$G_{dyn}(J,h) := \int_{[0,L]} \left\{ A_r \ddot{j}_0 \cdot h_0 + [I_r \dot{w} + w \times (I_r w)] \cdot J \right\} dS + G(J,h) \equiv 0 \tag{8}$$

where

$$G(J,h) := \int_{[0,L]} \left\{ N \cdot \Lambda^t [\frac{\partial h_0}{\partial S} - J \times \frac{\partial f_0}{\partial S}] + M \cdot \Lambda^t \frac{\partial J}{\partial S} \right\} dS - \int_{[0,L]} (\tilde{n} \cdot h + \tilde{m} \cdot J) dS \equiv 0 \tag{9}$$

and A_r and I_r are the sectional density of area and Inertia dyadic, respectively.

2.4 Time Integration

The Newmark's method is employed to integrate the linearized system obtained from the weak form. The novelty of the proposed approach lies in the treatment of the rotational part which relies crucially on the use of the discrete counterpart of the *exponential map* in the special group *SO(3)*. The basic problem concerning the discrete time stepping update is that given a configuration of displacements and rotation tensor $j_n := (d_n, \Lambda_n)$, linear and angular velocities and accelerations, (v_n, w_n), (a_n, a_n) in the time step n obtain the updated configuration in the time step $n+1$. The algorithm employed is summarized in the equations (10), (11) and (12) for the material description of the dynamic variables, Dt is the length of the time step. It is interesting to note that the updating procedure for the rotational part of the dynamic variables have to be carried out in the material description due to the fact that this configuration is time independent and the base point on the rotational manifold stays fixed [Makinen, 2001].

Implicit time stepping algorithm, (material description)

Translation	Rotation	
$d_{n+1} = d_n + u_n$	$\Lambda_{n+1} = \Lambda_n \exp[\Theta_n] \equiv \exp[q_n]\Lambda_n$	(10)
$u_n = (\Delta t)v_n + (\Delta t)^2[(0.5-b)a_n + b\,a_{n+1}]$	$\Theta_n = (\Delta t)W_n + (\Delta t)^2[(0.5-b)A_n + b\,A_{n+1}]$	(11)
$v_{n+1} = v_n + \Delta t[(1-g)a_n + g\,a_{n+1}]$	$W_{n+1} = W_n + \Delta t[(1-g)A_n + g\,A_{n+1}]$	(12)

Working on the linearized form of the equation (9) and employing standard techniques of the finite element method, is possible to obtain the discrete version of the system of equation [Simo et al, 1988]:

$$LG^{i}_{dyn\,n+1} = \sum_{I,J=1}^{N} h_I \cdot [P_I(j^{(i)}_{n+1}) + K_{IJ}(\Lambda_n, j^{(i)}_{n+1})\Delta j^{(i)}_{J,n+1}] = 0 \quad (13)$$

$$K_{IJ}(\Lambda_n, j^{(i)}_{n+1}) := M_{IJ}(\Lambda_n, \Lambda^{(i)}_{n+1}) + S_{IJ}(j^{(i)}_{n+1}) + G_{IJ}(\Lambda^{(i)}_{n+1}) + L_{IJ}(\Lambda^{(i)}_{n+1}) \quad (14)$$

where P_I and K_{IJ} are the vector of residual forces and the stiffness matrix respectively. The matrix K_{IJ} have contributions from the inertia, material and geometric terms M_{IJ}, S_{IJ} and (G_{IJ}, L_{IJ}) respectively. A step-by-step iterative Newton-Rapson scheme with a predictor–corrector method is employed to obtain the dynamic response of the system.

3. MULTIFIBERS BEAM ELEMENTS

The cross section of the beam is divided into an orthogonal non–homogeneous grid of cells as it is shown in Figure 2. This avoid the formulation of constitutive laws using sectional forces and displacements or moments and curvatures, which is the traditional way to solve the problem but valid only in certain particular cases [Oller and Barbat, 2006]. The sectional forces are decomposed fiber by fiber, in stress tensor which are corrected according with the constitutive laws given for the materials of the fiber and using the simple mixing theory to treat the resulting composite [Car, Oller et.al. 2000]. The corrected sectional forces and moments are then obtained by integration over the section area. The obtained sectional forces and moments are then used to compute the residual forces, in order to iterate for equilibrium if necessary (see Figure 2).

3.1 Constitutive Laws

In this work the concrete behavior is simulated employing a damage model based on the Kachanov theory for degrading materials. The model can take into account different properties for tension or compression. The steel reinforcements and stirrups are modeled employing elastic–plastic fiber behavior. Both models are thermodynamically sustainable avoiding the representation of the behavior of the materials in an approximated form based mainly on experimental studies. Strain localization is expected to occur in some members and therefore, a regularization of the dissipated energy is carried out at constitutive level to obtain objectivity in the response of the whole structure [Hanganu et.al. 2002].

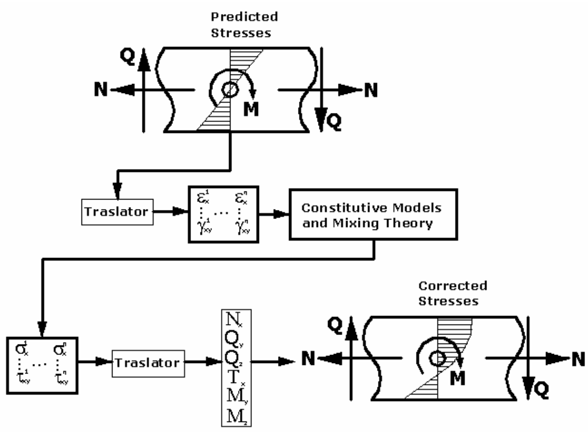

Figure 2: Iterative process at Integration point level in each section.

3.1.1 Damage constitutive model for concrete

The damage model has a rigorous but relatively simple formulation strictly based on thermodynamics [Simo and Ju, 1987]. The model is formulated in the material configuration with no temperature time variations. The free energy presents the following form:

$$y(e,d)=(1-d)y_0(e)=(1-d)(\frac{1}{2r_0}e^T s_0)=(1-d)(\frac{1}{2r_0}e^T C_0 e) \qquad (15)$$

where e is the strain tensor, $d\ (0\le d\le 1)$ is the internal damage variable, r_0 is the density in the material configuration, C_0 is the elastic constitutive tensor in the initial undamaged state. The fulfillment of the Clasius Planck inequality provide the constitutive law and dissipation, $\dot{\Xi}_m$, according to equation (16).

$$s=(1-d)C_0 e; \qquad \dot{\Xi}_m=-\frac{\partial\Psi}{\partial d}\dot{d}\ge 0 \qquad (16)$$

The damage yield criterion is defined in function of the free energy of the undamaged material as: $F=K(s_0)\sqrt{2r_0\Psi_0}-1$, where $K(s_0)$ is a function of the principal stresses and takes into account the ability of the model to consider different traction and compression properties [Barbat, et.al. 1997]. See Figure 3. Finally, the tangent constitutive tensor is given [Oller et.al. 1997] by

$$ds=C^D de=[(1-d)\underset{=}{I}-\frac{dG(\bar{s})}{d\bar{s}}s^0\otimes\frac{\partial\bar{s}}{\partial s^0}]C^0 de=(I-D)C^0 de \qquad (17)$$

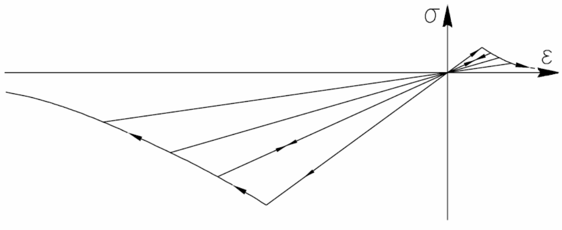

Figure 3: Local damage model with different tension–compression properties.

3.1.2 Constitutive model for steel reinforcements and stirrups

The steel of the bar reinforcements and stirrups is simulated by mean of employing a constitutive law for fibers. The fibers are treated as an orthotropic material with the steel's elastic modulus in the direction of the reinforcement and the concrete's elastic modulus in the other two directions. The Poisson coefficient is taken equal to zero to avoid introducing lateral interaction between concrete and steel reinforcements. After the yield criterion has been reached the plastic flux is oriented in the direction of the fiber [Car et.al. 2000].

3.1.3 Simple mixing theory for materials

The behavior of the composite is defined according to the fractional part of the total volume of the compounding substances (see Figure 4). It also assumes that in each material point all the components contribute with their own constitutive law in the assigned volume proportion. This allows combining materials with different constitutive behavior. In this work it is assumed that all phases in the mixture have the same strain field. The stress state of the composite is obtained starting from a hyper elastic model satisfying the dissipation condition of the second principle of thermodynamics, for *n* materials components, each of them with a volume proportion k_c, mass density m_c and free energy y_c, the stress is given by the equation (18) [Car et.al. 2000].

$$s = \sum_{c=1}^{n} k_c m_c \frac{\partial y_c}{\partial e} = \sum_{c=1}^{n} k_c m_c (s)_c \tag{18}$$

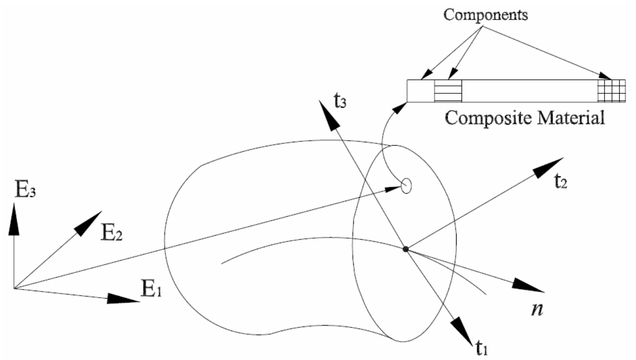

Figure 4: Each fiber of the section has assigned a composite material.

3.2 Energy Dissipating Devices

The energy dissipating devices are simulated using a rod element with only one Gauss integration point. The rotational degrees of freedom are released in both ends of the beam to obtain only relative displacements in the device. The constitutive law employed for dissipating devices corresponds to a bilinear plasticity, but any other one dimensional description can be employed, for example in Mata et.al. (2006), a constitutive description for elastomers to be employed in energy dissipating devices is given.

4. NUMERICAL EXAMPLES

In this section three numerical examples validating the proposed formulation for rods in the geometric and material nonlinear range are presented and explained. The first two examples correspond to the nonlinear elastic response of a rod in static and dynamic range. The third one corresponds to the study of a typical flexible and low damped concrete building, which is equipped with energy dissipating devices.

4.1 Elastic Large Bending of a Rod

This example presents the geometrically nonlinear analysis of a rectilinear cantilever beam with a bending moment, *M*, applied at the free end (Figure 5). See Ibrahimbegovic (1995) for more details. For the chosen data the values of the free end bending moments that turns the reference rectilinear line into the corresponding circle is 20*p*, if the applied moment is increased again the another smaller circle is formed at *M*= 40*p*. The beam is modeled employing 40 quadratic finite elements with two Gauss integration point to avoid shear locking in the response. It is possible to observe the ability of the model to predict the response of the elastic rod even for large displacements and rotations.

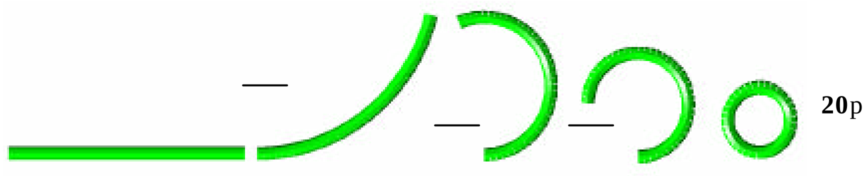

Figure 5: Nonlinear elastic bending of rod.

4.2 Dynamic Analysis of Beam: Large Rotations

The same beam of the previous example has been subjected to an imposed rotation of *p* at the clamped end. The dynamic response is obtained for a density mass of 3.0×10^{-5} Kg/cm^3, the result of the simulation allows to see the versatility of the formulation for predicting the complex configurations of the system during the motion. The rotational inertia terms are considered in the formulation allowing enhancing the prediction of the dynamic response for the case of large rotations, as it can be seen in Figure 6.

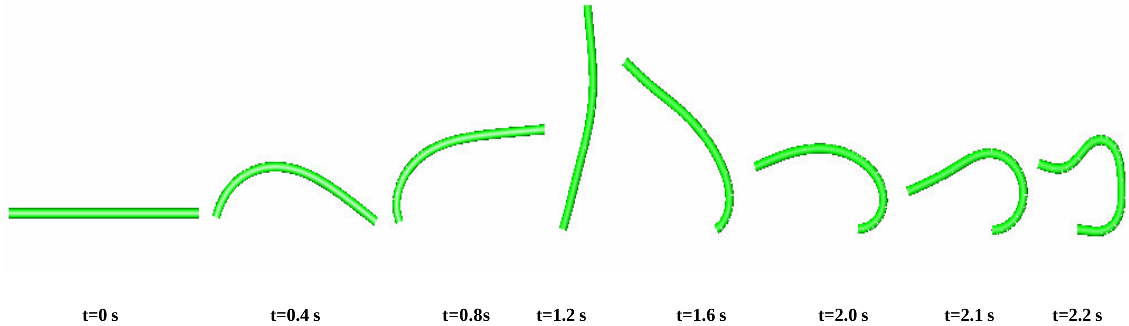

Figure 6: Nonlinear dynamic response of rod subjected to an applied rotation.

4.3 Nonlinear Seismic Response of Planar Frame

In this work the seismic nonlinear response of a typical concrete industrial building is studied. The building has a bay width of 20 m and 24 m of inter–axes length. The story high is 10 m. The concrete of the beam is H–50, (50 Mpa, ultimate compression), with an elastic modulus of 35.000 Mpa for the beam and H–30 for the concrete of the columns. It has been assumed a Poison coefficient of 0.2 for both concretes. The steel bar reinforcements considered in the study and the fiber discretization of the sections are those shown in Figure 7. The ultimate tensile stress for the steel is 510 Mpa. The dimensions of the columns are 60x60 cm^2. The beam has a variable section with an initial high of 80 cm on the supports and 120 cm in the middle of the span.

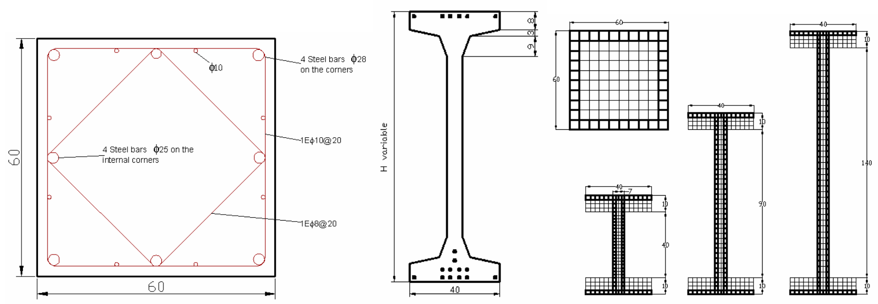

Figure 7: Columns and beam reinforcements and fiber model of the sections.

The permanent loads considered are 2000 N/m^2 and the weight of upper half of the closing walls, that is, 270,000 N. The employed acceleration record is the N–S component of the *El Centro* earthquake, 1940. The section of the energy dissipating devices was designed for yielding with an axial force of 300.000 N and for a relative displacement between the two ending nodes of 1.0 mm. The length of the dissipating devices is of 2.5 m (see Figure 8).

In Figure 9 it is possible to see the contribution of the dampers to reduce the displacements response. The obtained reduction is the order of 51% minimizing the P–Δ effects. The maximum acceleration shows a reduction of the order of 30% compared with the case where no devices are incorporated.

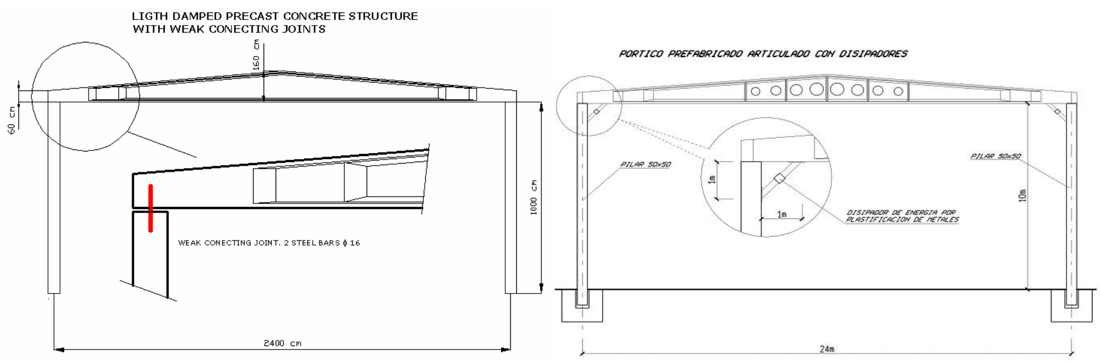

Figure 8: Precast industrial building without and with dissipators.

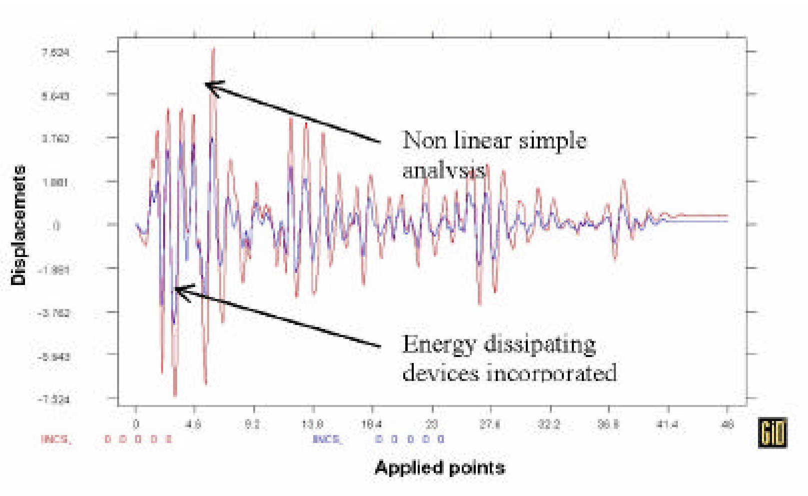

Figure 9: Displacements time history.

5. CONCLUSIONS

The geometrically exact formulation due to Vu–Quoc and Simo for beams and the use of appropriated constitutive laws for materials provides a useful tool to simulate the earthquake effects on concrete structures. A detailed study of the seismic response of structures requires taking into account the geometric and material nonlinear behavior of the structure for including the ductility demand on structural members, softening behavior, energy dissipation and the P-Δ effect. The plastic energy dissipating devices allows the improvement of the seismic behavior of flexible and low damped concrete structure studied in this work. From the presented studies it is possible to see that the use of plastic energy dissipating devices reduces the displacement response about 50% and the acceleration response 30% for the N–S component of the *El Centro* 1940 earthquake record.

6. ACKNOWLEDGMENTS

This work has been partially supported by: The University of Chile, Project FONDECYT nº 1970732; Ministerio de Ciencia y Tecnología, Spain, Project: "Numerical simulation of the seismic behavior of structures with energy dissipation systems", Contract nº: BIA2003 - 08700 - C03 – 02; and Integrated R&D Project of the EC "LessLoss–Risk Mitigation for Earthquakes and Landslides" funded by the European Commission, Directorate General of Research under the Contract nº GOCE-CT-2003-505448.

7. REFERENCES

Barbat A. H, Oller S, Oñate E and Hanganu A. (1997). *Viscous damage model for Timoshenko beam structures.* International Journal of Solids and Structures. Vol. 34. Nº 30. pp. 3953—3976.

Car J, Oller. S and Oñate E. (2000). *Modelo constitutivo continuo para el estudio del comportamiento mecánico de los materiales compuestos*. International Center for Numerical Methods in Engineering, CIMNE, Barcelona, Spain. PhD Thesis. (In Spanish).

Davenne L, Ragueneau F, Mazars J and Ibrahimbegovic. A. (2003). *Efficient approaches to finite element analysis in earthquake engineering.* Computers & Structures 81, 1223—1239.

Hanganu Alex D., Oñate E. and Barbat A.H. (2002). *A finite element methodology for local/global damage evaluation in civil engineering structures*. Computers & Structures 80, 1667—1687.

Ibrahimbegovic A. (1995). *On the finite element implementation of geometrically nonlinear Reissner's beam theory: three-dimensional curved beam elements.* Computer Methods in Applied Mechanic and Engineering. 122, 11—26.

Makinen J. (2001). *Critical Study of Newmark scheme on manifold of finite rotations*. Computer Methods in Applied Mechanic Engineering.191, 817—828.

Mata P., Boroschek R., Barbat A.H. and Oller S. (2006). *High damping rubber model for energy dissipating devices*. Journal of Earthquake Engineering, JEE, (accepted for publication).

Oller S. and Barbat A.H. (2006). *Moment-curvature damage model for bridges subjected to seismic loads*. Computer Methods in Applied Mechanic Engineering. (In Press).

Simo J.C, and Vu–Quoc L. (1985). *A finite strain beam formulation, Part I.* Computer Methods in Applied Mechanic Engineering. 49, 55–70

Simo J.C, and Vu–Quoc L. (1986). *A three dimensional finite strain rod model, Part II: Computational aspects.* Computational Methods in Applied Engineering. 58, 79—116.

Simo J.C, and Vu–Quoc L. (1988). *On the dynamic in space of rods undergoing large motions–A geometrically exact approach.* Computer Methods in Applied Mechanic Engineering. 66, 125—161.